

New Supersymmetric AdS_6 via T-duality

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We present a new supersymmetric AdS_6 solution of type IIB supergravity with $SU(2)$ isometry. This solution is the result of a non-Abelian T-duality on the known supersymmetric AdS_6 solution of massive IIA. The $SU(2)$ R-symmetry is untouched, leading to sixteen supercharges and preserved supersymmetry.

INTRODUCTION

Given a non-linear sigma-model (NLSM) with a target space-time geometry admitting an Abelian isometry, a well-defined prescription exists for gauging the isometry, integrating out the gauge field and producing the so-called T-dual sigma-model [1, 2]. Then, from the T-dual sigma-model it is possible to infer how the geometry changes under this T-duality transformation. The beauty of the gauging approach is that it is immediately generalisable beyond the Abelian case to both non-Abelian isometries, early accounts of which appear in [3–6], and more recently, fermionic isometries [7, 8] (see [9] for a recent review).

Non-Abelian T-duality has recently been upgraded to a symmetry of type II supergravity [10, 11], so new supergravity solutions can be generated from old ones. In contrast to Abelian T-duality, non-Abelian T-duality may not be regarded as a symmetry of String theory, and it has notable quirks. For instance, it is not clear how to constrain holonomies of gauge fields and show that the original and T-dual models have the same path integrals. However, a user-friendly description of the $SU(2)$ transformation [12] allows one to plug in a space-time with an $SO(4)$ isometry and generate a T-dual solution. In the process the chirality of the theory flips, i.e. from type-IIA to type-IIB and vice versa [10, 11]. Ref. [12] also shows that we can understand non-Abelian T-duality in terms of inert lower-dimensional theories that are invariant under the duality, just as in the Abelian case [13].

Separately, we have witnessed renewed interest in $D = 5$ superconformal field theories [14, 15] and a steady trickle of recent developments [16–19], some of which involve tests of the AdS_6/CFT_5 duality. In fact there is only one known supersymmetric AdS_6 solution [20], the existence of which was anticipated by [21], and orbifolds thereof [16]. Indeed, it has also recently been confirmed that this supersymmetric solution (and its orbifolds) is unique in massive type-IIA [22].

In fact, the dearth of supergravity solutions is not so surprising. Recall that the spinors in six-dimensions have eight-components. This means that AdS_6 , which admits both Poincaré and superconformal supersymmetries, preserves a total of sixteen supersymmetries. With

this amount of supersymmetry, it is not surprising that any space-time with an AdS_6 factor would be reasonably constrained.

The object of this note is to draw attention to another supersymmetric solution that can be constructed from the literature. While it is expected that Abelian T-duality on the $AdS_6 \times S^4$ solution of massive IIA produces another supersymmetric solution of type IIB with $SU(2) \times U(1)$ isometry, here we show that one also has the freedom to perform, following [12], an $SU(2)$ non-Abelian T-duality resulting in a new supersymmetric solution to type IIB with just $SU(2)$ isometry.

D4-D8 NEAR-HORIZON

The only known supersymmetric $AdS_6 \times S^4$ solution of massive IIA supergravity [23] arises as the near-horizon of D4-D8 [20].

The string frame solution is

$$\begin{aligned} ds^2 &= \frac{1}{4} W^2 L^2 [9ds^2(AdS_6) + 4ds^2(S^4)] , \\ F_4 &= 5L^4 (m \cos \theta)^{1/3} \sin^3 \theta d\theta \wedge \text{vol}(S^3) , \\ e^\Phi &= \frac{2}{3L(m \cos \theta)^{5/6}} , \end{aligned} \quad (1)$$

where m is the Romans' mass, L denotes the AdS_6 radius, W , the warp factor, is a function of θ , $W = (m \cos \theta)^{-1/6}$, and the metric on S^4 takes the form

$$ds^2(S^4) = d\theta^2 + \sin^2 \theta ds^2(S^3) . \quad (2)$$

While S^4 would have $SO(5)$ isometry, the θ -dependent warping means that this is broken to $SO(4) \sim SU(2)_G \times SU(2)_R$, where one $SU(2)_G$ is a global symmetry and the other an R-symmetry. In addition, as the range for θ is $0 \leq \theta \leq \pi/2$, instead of a whole S^4 , we only have half, and at one end-point of this range, $\theta = \pi/2$, the warp factor W blows up leading to a curvature singularity. In addition the string coupling e^Φ blows up.

NON-ABELIAN T-DUALITY

The non-Abelian dual of a general class of type-II supergravity solutions with isometry $SO(4) \sim SU(2) \times$

$SU(2)$, with respect to any of these $SU(2)$ subgroups, was given in [12]. Moreover, it was demonstrated that the original solution on S^3 , and the T-dual solution on the dual space $M_1 \times S^2$ (see below), may be reduced consistently to give the *same* theory in seven-dimensions [12], thus offering another perspective on the fact that non-Abelian T-duality is a symmetry of the equations of motion.

Recall from [12] that given a massive type-IIA solution of the form

$$\begin{aligned} ds_{IIA}^2 &= ds^2(M_7) + e^{2A} ds^2(S^3) , \\ F_0 &= m , \\ F_2 &= G_2 , \\ F_4 &= G_1 \wedge \text{vol}(S^3) + G_4 , \end{aligned} \quad (3)$$

where m is the mass, A is a scalar warp factor and the B -field, dilaton, Φ , and the n -form fluxes, G_n , just depend on the seven-dimensional space-time, the NS sector of the type-IIB supergravity $SU(2)$ T-dual is given by

$$\begin{aligned} ds_{IIB}^2 &= ds^2(M_7) + e^{-2A} dr^2 + \frac{r^2 e^{2A}}{r^2 + e^{4A}} ds^2(S^2) , \\ \hat{B} &= B + \frac{r^3}{r^2 + e^{4A}} \text{vol}(S^2) , \\ e^{-2\hat{\Phi}} &= e^{-2\Phi} e^{2A} (r^2 + e^{4A}) , \end{aligned} \quad (4)$$

where we have introduced hats to differentiate T-dual fields from those of the original solution. Observe that in the process of doing the $SU(2)$ transformation, one of the $SU(2)$ isometries is selected out and gets broken, leaving a manifest residual $SU(2)$ isometry in the form of the remaining two-sphere. In turn, (4) is a solution of the type-IIB equations of motion for any positive value of r . In order to fully clarify the nature of the space spanned by this variable we should resort to the sigma-model derivation of T-duality. However it is not clear how to extract global topological properties in the non-Abelian case [24]. The complementing general expressions for the RR fluxes post T-duality may be found in [12], and owing to their length, we omit them.

Although the equations of motion are guaranteed to be satisfied, more pertinent to our current discussion is the issue of preserved supersymmetry. From [12] we know that under an $SU(2)$ transformation from type IIB supergravity to massive IIA the Killing spinor equations may be mapped up to the gravitino variation in the r -direction. Interestingly, this single expression encapsulates all the information about the projection conditions on the Killing spinors of supersymmetry preserving T-duals. It is certainly expected that for transformations from massive IIA to type-IIB the supersymmetry conditions also simply boil down to one condition. Indeed, some work reveals this is the case and through the usual

rotation of the type-IIB Killing spinor

$$\eta = e^X \tilde{\eta} = \exp \left(-\frac{1}{2} \tan^{-1} \left(\frac{e^{2A}}{r} \right) \Gamma^{\alpha_1 \alpha_2} \sigma^3 \right) \tilde{\eta} , \quad (5)$$

where α_i , $i = 1, 2$ denote coordinates on the residual S^2 , one can demonstrate that if the original geometry is supersymmetric, then the T-dual geometry is also supersymmetric provided

$$\begin{aligned} \delta\psi_r &= e^X \left[\frac{1}{2} \not{\partial} A \Gamma_r - \frac{e^{-A}}{4} \Gamma^{\alpha_1 \alpha_2} \sigma^3 + \frac{e^\Phi}{8} \left(m i \sigma^2 \right. \right. \\ &\quad \left. \left. + e^{-3A} \not{G}_1 \Gamma^{r \alpha_1 \alpha_2} \sigma^1 + \not{G}_2 \sigma^1 - \not{G}_3 \Gamma^{r \alpha_1 \alpha_2} i \sigma^2 \right) \right] \tilde{\eta} \\ &= 0 . \end{aligned} \quad (6)$$

Note that η is further decomposed in terms of real Majorana-Weyl spinors

$$\eta = \begin{pmatrix} \epsilon_+ \\ \epsilon_- \end{pmatrix} . \quad (7)$$

T-DUAL AdS_6

Before performing an $SU(2)$ transformation on (1), we comment on the $U(1)$ T-duality in the same context. A natural $U(1)$ direction can be found by rewriting the metric on S^3 in terms of a Hopf-fibre over S^2

$$ds^2 = \frac{1}{4} [d\phi_1^2 + \sin^2 \phi_1 d\phi_2^2 + (d\phi_3 + \cos \phi_1 d\phi_2)^2] . \quad (8)$$

Here ϕ_3 labels the Hopf-fibre direction, T-duality on which has previously been discussed in the literature in [25], without commenting on the preserved supersymmetry. Indeed, the Killing spinors for the original $AdS_6 \times S^4$ solution with this parameterisation of the S^3 take the form

$$\eta = (\cos \theta)^{-1/12} e^{-\frac{\theta}{2} \Gamma^\theta \sigma^1} e^{-\frac{\phi_1}{2} \Gamma^{\phi_3 \phi_2}} e^{-\frac{\phi_2}{2} \Gamma^{\phi_2 \phi_1}} \tilde{\eta} , \quad (9)$$

where $\gamma = \Gamma^\theta \phi_1 \phi_2 \phi_3$ and $\tilde{\eta}$ denotes the Killing spinor on AdS_6 . The Killing spinor is subject to a single projection condition

$$[\sin \theta \Gamma^\theta \sigma^1 + \cos \theta \Gamma^{\theta \phi_1 \phi_2 \phi_3}] \eta = -\eta , \quad (10)$$

so we have sixteen supersymmetries, the minimum required for a supersymmetric AdS_6 geometry. Furthermore, as is evident from the explicit form of the Killing spinor, it is independent of ϕ_3 , so that when one performs the Abelian T-duality one expects no supersymmetry to be broken. By explicitly working out the Killing spinor equations for the Abelian T-dual one can also confirm this to be the case. So supersymmetric AdS_6 geometries in type-IIB certainly exist.

The main result of this letter now follows. The $U(1)$ Hopf-fibre T-duality produces a supersymmetric T-dual because we are simply picking out a $U(1)$ direction from the $SU(2)$ global symmetry. Therefore, in the process of doing the T-duality, the $SU(2)$ R-symmetry is untouched. Now, we also have the freedom to do an $SU(2)$ T-duality using the full global symmetry. Again the rational is the same; as we do not touch the R-symmetry we are guaranteed to produce a supersymmetric solution. So cranking the handle, one arrives at

$$\begin{aligned} d\hat{s}^2 &= \frac{1}{4}W^2L^2 [9ds^2(AdS_6) + 4d\theta^2] \\ &\quad + e^{-2A}dr^2 + \frac{r^2e^{2A}}{r^2 + e^{4A}}ds^2(S^2) , \\ \hat{B} &= \frac{r^3}{r^2 + e^{4A}}\text{vol}(S^2) , \\ e^{-2\hat{\Phi}} &= e^{-2\Phi}e^{2A}(r^2 + e^{4A}) , \\ \hat{F}_1 &= -G_1 - mrd r , \\ \hat{F}_3 &= \left[-\frac{r^3}{r^2 + e^{4A}}G_1 + \frac{mr^2e^{4A}}{r^2 + e^{4A}}dr \right] \wedge \text{vol}(S^2) , \end{aligned} \quad (11)$$

where we have introduced the following

$$e^A = \frac{WL \sin \theta}{2}, \quad G_1 = \frac{5}{8}L^4(m \cos \theta)^{1/3} \sin^3 \theta d\theta . \quad (12)$$

At $\theta = 0$, just as with the Abelian T-dual, there is a curvature singularity and $e^{\hat{\Phi}}$ blows up. This is in addition to the singularity at $\theta = \pi/2$ inherited from the original solution.

We are now in a position to plug this solution back into (6), the only independent Killing spinor equation post T-duality, to identify the projection conditions on the Killing spinor. In the process, one encounters a single projection condition

$$[\cos \theta \Gamma^{\theta r \alpha_1 \alpha_2} \sigma^3 - \sin \theta \Gamma^{\theta r} i \sigma^2] \tilde{\eta} = -\tilde{\eta} , \quad (13)$$

thus showing that supersymmetry is preserved. Moreover, by employing the redefinitions

$$\tilde{\epsilon}_+ = \Gamma^\theta \epsilon_+, \quad \tilde{\epsilon}_- = \epsilon_-, \quad \Gamma^{r \alpha_1 \alpha_2} = -\Gamma^{\phi_1 \phi_2 \phi_3} , \quad (14)$$

one can recover the original projector (10).

DISCUSSION

While it can be rationalised at some level, i.e. we are not touching the R-symmetry, this indeed is a striking result. To appreciate this, recall that even for flat space-time, the $SU(2)$ T-duality transformation we have employed here breaks supersymmetry by one half [12]. So, in the original warped supersymmetric $AdS_6 \times S^4$ solution of massive type-IIA, we have found the first example of a non-Abelian T-duality transformation where supersymmetry is preserved. It turns out that the example

presented in this paper is however not unique. Other examples based on the Klebanov–Witten and Klebanov–Strassler $\mathcal{N} = 1$ backgrounds, for which supersymmetry is also preserved under non-Abelian T-duality, have also been constructed in [26].

A related question concerns the AdS/CFT interpretation. The identification of the dual SCFT for the non-Abelian T-duality transformation is a long-standing problem and the jury is certainly out on whether one exists, and if it does, whether it is the same SCFT, or indeed a different theory. In the process of doing the $SU(2)$ T-duality in the $AdS_6 \times S^4$ context, the $SU(2)$ global symmetry is completely broken, leaving just the R-symmetry. For the Abelian T-duality the isometry is also reduced, but there we are confident that the theory does not change. In fact, the Cartan of the isometry group remains the same. On the other hand, in the $SU(2)$ T-dual such Cartan subgroup seems different than that of the original background suggesting that the dual theory – if it exists – would be different. In any rate, further checks are certainly required. A careful analysis of the dual SCFT is underway in order to try to elucidate some of its properties [27]. The SCFTs associated to the non-Abelian duals of the Klebanov–Witten and Klebanov–Strassler backgrounds are being studied in [26].

Moreover, now that we have two distinct solutions in type-IIB, it may be an opportune time to build on the work initiated in massive IIA [22] and classify the supersymmetric solutions in this setting also. A priori we will have at least two branches, one with $U(1)$ T-dual and the other with the $SU(2)$ T-dual.

Finally, another interesting direction for study concerns the KK reduction [28] from massive IIA on S^4 to Romans' F(4) supergravity [29]. In [12] it was shown that there was a consistent truncation to $D = 7$. The only terms of the KK reduction inconsistent with $SU(2)$ T-duality as described in [12] are the $SU(2)$ gauge fields. So, as it stands, any solution to Romans' theory now also uplifts to a solution to type-IIB provided the $SU(2)$ gauge fields are not excited. In this sense, here we are simply discussing the supersymmetric AdS_6 vacuum. We can think of putting the gauge fields back in if we gauge the residual $SU(2)$ R-symmetry of the non-Abelian T-dual. This all echoes well with the conjecture [30] that gauging the R-symmetry always leads to a consistent reduction.

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- [1] T.H. Buscher, Phys. Lett. **B201** (1998) 466; Phys. Lett. **B201** (1988) 466.
 - [2] M. Rocek and E.P. Verlinde, Nucl. Phys. **B373** (1992) 630 [hep-th/9110053].
 - [3] X.C. de la Ossa and F. Quevedo, Nucl. Phys. **B403** (1993) 377 [hep-th/9210021].
 - [4] A. Giveon and M. Rocek, Nucl. Phys. **B421** (1994) 173 [hep-th/9308154].
 - [5] K. Sfetsos, Phys. Rev. **D50** (1994) 2784 [hep-th/9402031].
 - [6] E. Alvarez, L. Alvarez-Gaume and Y. Lozano, Nucl. Phys. **B424** (1994) 155 [hep-th/9403155].
 - [7] N. Berkovits, J. Maldacena, JHEP **0809**, 062 (2008). [arXiv:0807.3196 [hep-th]].
 - [8] N. Beisert, R. Ricci, A. A. Tseytlin and M. Wolf, Phys. Rev. **D78** (2008) 126004 [arXiv:0807.3228 [hep-th]].
 - [9] E. Ó Colgáin, arXiv:1210.5588 [hep-th].
 - [10] K. Sfetsos and D.C. Thompson, Nucl. Phys. **B846** (2011) 21 [arXiv:1012.1320 [hep-th]].
 - [11] Y. Lozano, E. Ó. Colgáin, K. Sfetsos and D.C. Thompson, JHEP **1106** (2011) 106 [arXiv:1104.5196 [hep-th]].
 - [12] G. Itsios, Y. Lozano, E. Ó Colgáin and K. Sfetsos, JHEP **1208** (2012) 132 [arXiv:1205.2274 [hep-th]].
 - [13] E. Bergshoeff, C.M. Hull and T. Ortin, Nucl. Phys. **B451** (1995) 547 [hep-th/9504081].
 - [14] N. Seiberg, Phys. Lett. **B388** (1996) 753 [hep-th/9608111].
 - [15] D.R. Morrison and N. Seiberg, Nucl. Phys. **B483** (1997) 229 [hep-th/9609070].
 - [16] O. Bergman and D. Rodriguez-Gomez, JHEP **1207** (2012) 171 [arXiv:1206.3503 [hep-th]].
 - [17] D.L. Jafferis and S.S. Pufu, arXiv:1207.4359 [hep-th].
 - [18] H.C. Kim, S.S. Kim and K. Lee, arXiv:1206.6781 [hep-th].
 - [19] O. Bergman and D. Rodriguez-Gomez, arXiv:1210.0589 [hep-th].
 - [20] A. Brandhuber and Y. Oz, Phys. Lett. **B460** (1999) 307 [hep-th/9905148].
 - [21] S. Ferrara, A. Kehagias, H. Partouche and A. Zaffaroni, Phys. Lett. **B431** (1998) 57 [hep-th/9804006].
 - [22] A. Passias, arXiv:1209.3267 [hep-th].
 - [23] L.J. Romans, Phys. Lett. **B169** (1986) 374.
 - [24] E. Alvarez, L. Alvarez-Gaume, J.L. F. Barbon and Y. Lozano, Nucl. Phys. **B415** (1994) 71 [hep-th/9309039].
 - [25] M. Cvetič, H. Lu, C.N. Pope and J.F. Vazquez-Poritz, Phys. Rev. **D62** (2000) 122003 [hep-th/0005246].
 - [26] G. Itsios, C. Nunez, K. Sfetsos and D.C. Thompson, to appear.
 - [27] Y. Lozano, E. Ó Colgáin, D. Rodríguez-Gómez and K. Sfetsos, to appear.
 - [28] M. Cvetič, H. Lu and C.N. Pope, Phys. Rev. Lett. **83** (1999) 5226 [hep-th/9906221].
 - [29] L. J. Romans, Nucl. Phys. **B269** (1986) 691.
 - [30] J. P. Gauntlett and O. Varela, Phys. Rev. **D76** (2007) 126007 [arXiv:0707.2315 [hep-th]].